Mathematic Modeling and Its Role in Operational Research

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ABSTRACT

Relation between mathematics and economic activity has a dual character: mathematics feed from economic and social environment through different kinds while economic sciences, including leading science, are mathematized in a fast rhythm.
Under the conditions of the dynamic of contemporary economic life, is impossible to have the decisions adopted only by the means of intuition and usual judgment. For this, an important aid is given by mathematic-statistic methods, that is, operational research. Grace of operational research, the usual reasoning, which is always more or less empiric and intuitive, is filled with mathematic reasoning, rigorous, exact.
For have an overview over the object of operational research applied in economy, we consider to shortly study in this work, how appeared and developed management and leading branches and also the links between these branches. We also try to put in evidence the role of mathematical modeling in operational research.

Keywords: modeling, operational research, cybernetic, informatics, system analysis

1. OPERATIONAL RESEARCH AND RELATED BRANCHES

Operational research is one of branches which appeared to the end of the first half of XX century and has spectacularly developed, especially during last years, in a strong link with a series of other branches of management and leading like cybernetic, informatics and system analysis.
The concept of "scientific organization" shaped to the end of XIX century and the beginning of XX century, considers productive unity as a mechanism wherein men, helped by machines, work in an almost wholly determinism, based on some disposals which act hierarchically, in conformity with some competencies rigorously defined. The main representatives of the beginnings of scientific organization, who form so-called "classical school", established for the firs time a series of rules of scientific leading. Among these, there is also the well-known (and still actual) principle of exception, principle of organizational training, principle of rigorous definition of tasks, hierarchic organization principle (Staff and Line) and so on.
Between the concepts used by classical school, neither there is information, nor decision. Leading of social-economic "mechanism" comes back (in the latest, through the running of hierarchic pyramid steps), always, to a unique decision core, wherefore information are supposed, so they are, wholly and instantly usable, whit no kind of restriction (of time, space, sending and storing technique etc.).
However its limited perspective, classical school has the great merit of clearing an unknown domain. The pioneers of scientific organization (Taylor, Gantt, Fayol) and the other representatives of classical school put for first time the problem of rational approaching of the mechanism of enterprise operation. Most of the ideas of classical school have been criticized by the representatives of different schools which have further developed in management sciences, producing, how we'll see further, theories more and more abstract and complex. It merit to show that in the sixth decade, as a reaction against theorization excess, a so-called neoclassical school has developed, having as aim the returning in practice.
In the decades which follow after apparition and development of classical school, informational-decisional problems show their presence acuter and acuter, in the rhythm of the growth of dimensions and complexity of social-economic organizations and searches itself empiric solves,

the most times not to the level of needs. There are often established parallel and overflow (redundant) informational circuits and out of data official (formal) fluxes, a non-formal circulation is developed, sometimes more efficient but having strict local character. In continuous decision problems must impose routine, well-feeling, skill or even improvisation.

During the time after the First World War, could be observed, grace of those empiric solves, big differences, in what concerns competitiveness, between economic unities with equal or similar management features and technical givens. Made analyzes, led to a first inclusion within the area of the research concerning management and leading problems, of informational-decisional features, ignored till then and also of human relations features. Management and leading problematic is considerably broadened and the words “management” (as practice activity) and “management science” begin to run with more and more authority. This while is dominated by “behavior school” which put in the core of its studies the rigorous observation of human behavior during the motivation process which determines groups’ unity.

The important differences between behavior and classical school refer especially to features like: decisions decentralization, promotion of trust between the members of a group (and authority neglect) with the accent on responsibility, not on control.

Starting from the fifth decade of XX-th century it appears a phenomenon which promotes information and decision among main elements of nowadays.

To this phenomenon, firstly contributes the extraordinary growth of structural and functional complexity of economic organizations. The processes of agglomeration-integration, the apparition of organizational structures having productive activities on very large geographical surfaces (and also with many problems concerning products selling-off), rising of equipment technicality level and corresponding to, a strong specializing of professions – are only a few of the features of this complexity of modern productive units. As a consequence of this status, it appears an extraordinary growth of the quantity of information held and managed in production units, pointed also by the formulation of more difficult conditions in what concerning information quality (its pertinence and operativity). Besides goods production, it appears a more and more important production of information, information becomes a product or ware which can be neglected, arriving, besides the services, objective of some specialized organizations.

In what concerning decision processes, for the first time there is rigorously and on large scale put on the problem and finding optimum or near optimum solutions, in the large amount of organizational and management problems. We can consider that all these changes brought to a true informational-decisional revolution in management and leading domain and, as a consequence, to the apparition of modern scientific management.

The main branches concerning the leading, which have appeared during this period are: operational research, cybernetic, informatics, management psycho-sociology, and general theory of systems.

• **Operational research,** which can be shortly defined as branch of decision optimization by the means of mathematical modeling, appeared during the Second World War. Considered by ones as representing mathematical school in the branches of management and leading, operational research is firstly characterized through model elaboration process mathematized as a rule, which describe economic processes wherefore it follows to take decision as advantageous as possible. Pointing the importance of modeling in operational research, we wholly dedicate it next item.

• **Cybernetic** is the science which studies the management and regulation of complex systems. Among these characteristic attempts for improving used methods during last decades within management and leading sciences, besides massive use of mathematical procedures and electronic computers, there is also the use of system-cybernetic concept.

It can be defined as system, every section of reality wherein there is identified an assembly of phenomena, objects, processes, concepts, beings or groups connected through a manifold of mutual relations, and also with near environment, and which act together for realize clearly defined objectives. The manifold of elements and relations between these, and also

of the relations between components and assembly, make system structure. The manifold of the characteristics of a system, at a given moment, draw its status.
For system analysis considered together, there is purposed the concept of “black box” which represents the system studied as a whole, excepting its internal processes. The black box receive impulses from the environment (system “inputs”) and working out these impulses, turn they into actions over the environment (system “outputs”).
The mechanism of turning inputs into outputs can be described by the helping of transfer functions, which have different shapes, particular, as how the system is.
The system becomes cybernetic when the regulation appears (reverse connection, the feed-back) that is an intervention over the inputs for maintain the outputs to the level of some desired parameters of objective.
There is known that analytic expression of transfer functions and of regulation mechanism leads to very diverse and in most cases very complex mathematical shapes.
The whole of economy can be viewed as a system whose elements (social-economic organizations of different sizes) are inter-correlated through material and informational fluxes and have a behavior oriented to reaching exact objectives. At their turn, the organizations, which are components of the system as a whole, can be considered systems, the division being able to continue till finding elementary and indivisible components.

The scope of cybernetic-systemic research applied to social-economic reality is the surprising of systems behavior, one of the ways to describe this behavior being the founding of transfer expressions and of regulation mechanism. The adoption of cybernetic-economic perspective in social-economic sciences is an outstanding theoretical gain and it is very probably to assist next years to the binding of a complete and whole cybernetic-systemic theory, applied to social-economic reality at a large scale.

• **Informatics** can be defined as the branch of data working out, by the means of self-acting equipments of conditioning. The main problems which can be considered as informatics belongings are: data collection, data preparing, their encryption, their sending, data working on equipments, their storage and storing.
The problem of explosive development of informatics and its role in economy, administration, space research, military strategy, science, teaching, is well-known also by non-experts. We’ll show only that, from a few electronic computers, and few informatics experts, in 1945, nowadays there are reached in the whole world, millions of computers and experts. .

• **Psycho-sociology of management** appeared as a new directing in management branches around the year 1950. St. March, F. Simon and other representatives of so-called “psycho-sociological school”, mainly approach the problem of the influence of psychological and sociological factors in decision behavior. Taking decisions depends not only on rational criteria but also on the kind of stimuli perception, depending on the position of decision man and on the rapports with the other members of the group. With other words, howsoever it would make appeal, in economic organisms’ management and leading, to methods and equipments of big finesse and technicity, in the latest, humans are those by who depends efficient functioning of the system. For this reason it must be studied particular reactions and the relations between the persons of the group.

• **General System Theory (GST)**, strong linked to cybernetic, proposes a perspective to synthesize viable ideas of different orientations in management and leading sciences.

Relied on these facts, Forrester builds a drawing procedure of the behavior of an enterprise, which uses cybernetic, informatics, psycho-sociological methods and also mathematical modeling procedures. There are also used physical and technical analogies (for example, fluxes are checked hydraulically) and the simulation is used as a basic procedure in system behavior description.

The GST ideas and procedures, outstanding grace of their complexity, are running to methodological sedimentation and practical experimentation. The very most of the phrases enumerated above and which are on the base of Forrester’s theory are explicitly or implicitly regained also on the base of practical methodologies of systemic analysis. The concepts of informational flux and decisional process are the main in systemic analysis like in GST, and the following up of the mechanism of turning inputs into outputs is the main object of system analysis as of GST. The procedure used by systemic analysis is no more a mathematical one but based on explicit and qualitative description, of informational-decisional processes. Additionally, in system analysis practice, the same time with informational processes drawing and especially the projection of those decisional, there is followed their improving, therefore there are pointed optimum criteria. In this action of efficient projection of informational-decisional process, systemic analysis often calls operational research procedures and informatics techniques. In system analysis there are recent attempts in this sense.

2. MODELING ROLE IN OPERATIONAL RESEARCH

The concept of "model" so much used in modern science, is relatively new but modeling method is as old as human studies for scientific knowledge. We can consider the model is an isomorphic shape of reality, that offering an intuitive and however rigorous image, in the sense of logical structure of studied phenomenon, facilitates the development of some links or rules impossible or very hard to find on other ways. Sciences men of all times have used "models" in the most variety of scientific knowledge branches. Almost till now, they used modeling without use that term. In mathematical models elaboration, economic theory has a very important role because it shapes the categories, objective concepts and rules of economic reality. Only relying on economic theory, mathematical models can represent clearly economic phenomena.

Model, as scientific knowledge instrument, is used in very many theoretical and practical branches. Without claim to make a rigorous classification of model kinds, we'll show that they can be verbal-descriptive – used in all mathematized branches, mathematical models, physic-analogical models (of static or dynamic lays-out kind) graphical models etc.

Within economic sciences, especially in management and organization branches, models are used in all existent scale of types. But during last decades, there is shaped more and more the tendency of using in these branches especially of mathematical models especially grace of their capacity to rigorously condense the essential but also of their possibility to be programmed by the helping of electronic computers, featuring together an instrument of scientific investigation at an unknown power till now, a prodigiously "prolongation" of human intelligence.

A methodological systematization of mathematical models used in the branches of social-economic management and leading sciences would be at risk, viewing the spectacular movements within these branches and, additionally, would have a pure scholastic character, without real theoretical or practical utility. Hence, we'll limit in the followings to enumerate the main types of mathematical models known in this domain.

In rapport to the area of studied domain models which describe economic reality can be:

- *Macroeconomic models* – those which refer to national economy, to branch (under-branch) or to the economy of a big territory (a county, a certain industrial or agricultural area etc.) and microeconomic ones – at level of enterprise, plant, company, area of fabrics etc.
- *Cybernetic-economic models* follow to study the rapport between inputs and outputs within an economic organism with putting in evidence of regulation phenomena that determine the system good operation. The most of cybernetic-economic model are macroeconomic.
- *Econometric models* describe the behavior of economic organisms by the helping of a system of equations wherein numerical elements are statistically determined. These models are also macroeconomic as usual.
- *Simulation models* try to establish the operation kind of some macro - or microeconomic organisms through granting of random value combinations to the independent variables which describe the processes. From the "reading" of the values whereon this way the dependent variables get, there are gained important sizes in studied process.

- *Systemic model* have as objective the surprising of the ensemble of particularities from an economic organism (for example, in Forrester models there is considered that from the identification of the six characteristic fluxes, it can know the behavior of the system as a whole).
- *The models of operational research* are characterized through the searching of an optimal or near optimal solution, for studied phenomenon. The models of operational research rely on a big variety of mathematical procedures and have application of macro- but especially at microeconomic level. They represent the main instrument for decision optimization in system analysis.

The above topology is very relative, between mentioned groups, being frequent similarities and overlaps. Thus, economic models are often of cybernetic kind; simulation is used in almost all kinds of mathematical models; models of operational research can be used in systemic description of an organism etc.

We'll examine, in the following, practical procedures of elaboration and use of mathematical models in management and leading branches.

Firstly, it must be pointed that modeling activity, for be efficient, must be always developed within system analysis, that is, as a moment of drawing stage of the new system. A series of operations which are developed within system analysis before this moment, have a preparing character for making modeling, and others, further it, are necessary for application in practice of elaborated models.

We'll further show which are the main phases of the elaboration of a mathematical model in a social-economic management-leading problem, taking care to evidence how these phases join to other operation of system analysis.

First phase of modeling, which has a preparing character, is reality knowledge in studied organism, for improving informational-decisional mechanism description of decision processes logic, besides the considerration of future system objectives, are the main elements of knowledge of reality necesarry for modeling.

The second phase of modeling is the properly building of the model. This operation, in the very most cases in practice, consists of the application of a classical modeling instrument, chosen from the extreme diverse scale which operational research theory put for us. In such situations, analyst's skill consists of correspondence establishment between reality and modeling instrument known in dedicated literature. There are also cases when such correspondence can't be established, the analyst being obliged to elaborate new models. These can be of two kinds: a) combinations of classical models from theory domain and b) new properly models. In the first case, all is reduced to the good knowledge of reality and theory, whereat a skill portion in method combination must be added. In second case, it says about original creation. The elaboration of the really original mathematical model claims, besides deep knowledge of reality which follows to be modeled, a very solid mathematical culture, imagination and skill. As how it results from dedicated literature, there is a big variety in mathematical structure, and model logic, from very simply models, non-axiomatized, how are those of linear programming, to combinatorial ones, in problems of graph theory, critical way analysis and operative programming of production and till very finesse models, showed axiomatized like those of utility or group decisions.

Clearly, elaboration in axiomatized form of a model is a superior stage in modeling process, which, unfortunately, can't be always reached in practice.

An axiomatized model (axiomatic system) contains:

- *System axioms*, representing phrases explained in mathematical form, very few as usual, which contain some truths of big generality concerning the phenomenon which is modeled, so general than all objective and particular ascertainments, will be able to be deduced from those general;
- *Inference rules* representing rigorous prescriptions, the only admitted into the system, where through it passes from axioms to theorems or from already demonstrated theorems to other new ones.
- *Theorems*, these are more or less particular phrases, mathematically explained,

deducted through inference rules step by step, from postulates and which explain properties of modeled phenomenon;

When in axiomatic modeling process the concepts which follow to be used are explained in limitative kind, therefore a list of mathematical notions and operations admitted in system is given from beginning, there is gained a superior shape of axiomatic system named formal system. Formal systems are still very little used in science and so, less used in economic management and leading sciences.

Axiomatization and, at last analysis, formalization, represent the future in mathematical modeling, grace of exceptional rigor they put in, considerable decay of intuition and arbitrary elements that, however much less than in non-mathematical models, are still present in axiomatized mathematical modeling..

The third phase of modeling is the model confrontation with reality, and eventually, its experimentation. This phase is realized within system implementation which can be considered the forth and the last phase of modeling.

3. CONCLUSIONS

One of the main characteristics of operational research methods is that some problems of operational research can be theoretically regarded, as pure mathematical ones. As economic analysts, we are firstly interested in the link of mathematical models to reality, the capacity of those methods and models to reflect economic reality and to effectively contribute to the improving of the practice of management decisions adoption.

Historically, some of operational research problems appeared, rightly, under pure mathematical aspect, many years before the apparition of organized activity and the name of operational research. Therefore, some notions of graph theory are known since more than a century, waiting theory has the origin in some Erlang's works since second decade of XX century, and stock theory appeared towards 1930.

As self-contained branch, operational research hardly appeared during the World War II, through the making of complex teams (mathematicians, engineers, economists, biologists, psychologists and so on) having tasks of optimization of decisions concerning actions preparatory for military operations. After the war, the teams so formed have rapidly re-profiled for peaceful activities. Having a spectacular development last three decades, theoretical and especially practical activities in the branch of operational researched, nowadays reached to worldly train hundreds of thousands of experts.

Now, it can't be considered the leading of an important technical-economic activity without calling operational research methods, of course, together to the other modern techniques like informatics, system analysis and so on.

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